Delta Hedging:
Managing Market Risk with Derivatives

September 2023

Sven Peter Kooi
Thomas van den Berg

Summary

This article introduces key concepts in options trading, focusing on the Delta Greek and Delta Hedging. Greeks gauge option price sensitivity to factors like stock price and time-to-maturity. Delta, a primary Greek, shows how option prices change with stock price shifts. Call option Deltas range from 0 to 1, representing positive correlation, while put option Deltas range from -1 to 0, showing a negative correlation.

Delta Hedging is a strategy to manage risk caused by stock price fluctuations. It aims for a balance (Delta neutrality) between option and stock positions to counter risk. Dynamic Delta Hedging involves frequent adjustments, mitigating risk but considering trading costs and time constraints.

How can Amsshare support

The information provided in this article is well-known by Amsshare. Hence, Amsshare can support firms with projects within this area. For example, Amsshare is able to program Hedging Simulations by using Excel's programming language called Visual Basic for Application (VBA). The simulations can be based on different option pricing models, such as the Black-Scholes model, Heston model, etc. Then we can program a dashboard to calculate the hedging positions for the preferred Hedging Strategy.

Introduction

In Finance, we have tools called the Greeks that help us understand risks in options trading. This article discusses the five most common Greeks. However, only the Delta Greek will be explained in more detail, as the primary focus of this article is directed towards the explanation of Delta Hedging. It's important to understand the mechanics of the Delta Greek in order to understand the mechanics of Delta Hedging. The other Greeks can also be used for Hedging, but this is not covered in the article.

Greeks

The 'Greeks' of options measure the sensitivity of option prices with respect to the input parameters of the options. Input parameters of options are the determinants of the option price. Examples of these determinants are the stock price (the underlying), strike price, volatility, time to maturity and the interest rate. For example, if the stock price goes up, then the price of the corresponding call option will also go up and the price of the corresponding put option will go down. The degree of sensitivity is measured in the 'Greeks'. The five most common Greeks are:

1. Delta: this is the first partial derivative of the option price with respect to the stock price.
2. Gamma: this is the second partial derivative of the option price with respect to the stock price.
3. Theta: this is the first partial derivative of the option price with respect to the time-to-maturity.
4. Vega: this is the first partial derivative of the option price with respect to the volatility.
5. Rho: this is the first partial derivative of the option price with respect to the interest rate.

The Delta Greek

The Delta measures the sensitivity of the option price with respect to the stock price. This shows what the impact of a stock price decrease or increase is on the option price. In this article, the focus is primarily on call and put options. The Delta of a call option is always between 0 and 1. This relationship is positive, because the direction of a stock price change is the same as the direction of the change in call option price. A Delta of 0 means that a change in stock price does not have any effect on the option price. A Delta of 1 means that a change in stock price has a 1-to-1 relationship with a change in the option price. For example, when the stock price increases by 2%, then the call option price also increases by 2%.

The Delta of a put option is always between -1 and 0. This relationship is negative, because the direction of a stock price change is the opposite of the direction of the change in put option price. When the stock price increases, the put option price should stay the same or decrease. This is because one can sell the stock in the market for a higher price (because the stock price increased), so the value of a put option decreases. A Delta of -1 means that the change in stock price has a negative 1-to-1 relationship with the change in the option price. For example, when the stock price increases by 2%, then the put option price decreases by 2%.

Figure 1. *Delta Call option*

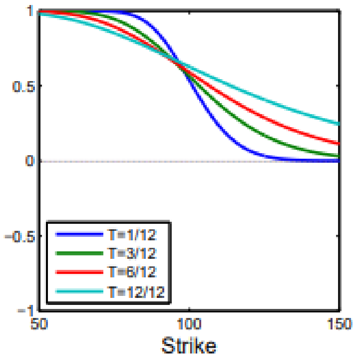
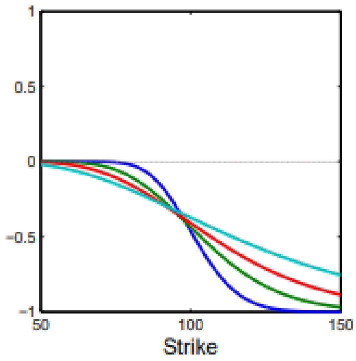


Figure 2. *Delta Put option*



Source: *Seegers, N. (2023), Derivatives*

Figure 1 shows the graphical representation of the Delta of a call option. The X-axis is the strike price of the option and the Y-axis is the Delta. The strike price of 100 is the At-The-Money (ATM) location of the call option, meaning that the stock price is also at 100. This means that a strike price below 100 (the left side of the graph) is the In-The-Money (ITM) location of the call option, and the strike price above 100 (the right side of the graph) is the Out-The-Money (OTM) location of the call option. There are different coloured graphs in the figure. These different colours represent different time-to-maturities of the call option. So, it can be concluded that (1) the different strike prices and (2) the different time-to-maturities impact the Delta of the call option. Around ATM, the graph of the 1/12 term option (time-to-maturity of 1 month) is much steeper than the 12/12 term option (time-to-maturity of 1 year). This indicates that the 1/12 term option reacts more heavily on a change in strike price than the 12/12 term option. The reasoning behind this is that the 1/12 term option has less time to end up ITM or OTM than the 12/12 term option. For all time-to-maturity options, it can be concluded that the Delta is the most sensitive around ATM.

For each time-to-maturity call option the Delta will result in a 1 when the strike price decreases. The Delta of 1 means that there is a 1-to-1 relationship with the underlying, which is the stock. So, a Delta of 1 occurs when the call option is exactly the same as the stock. On the other hand, for each time-to-maturity call option the Delta will result in 0 when the strike price increases. If this happens, it means that the call option is deep OTM. When the call option is deep OTM, a change in the underlying does not have an impact on the call option price, meaning that the Delta is 0.

Figure 2 shows the graphical representation of the Delta of a put option. If you mirror the explanation of the call option from above, you have the interpretation of the Delta of a put option. In this case the left side of the ATM location is the OTM location, and the right side of the ATM location is the ITM location. Besides, one can use the Put-Call-Parity to calculate the Delta of a call option when you know the Delta of the put option, or vice versa. However, the contract specifications (such as time-to-maturity, underlying, risk-free rate, etc.) need to be the same. The formula for this is:

$$\text{Delta Call option} = \text{Delta Put option} + 1$$

Delta Hedging

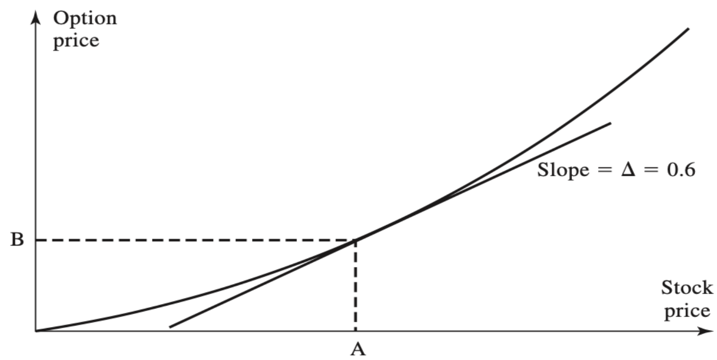
As discussed, the Delta is the sensitivity of the option value with respect to the underlying. As the value of stocks move over time, so does the value of an option. Since the walk of a stock price is hard to predict is it interesting for investors/asset managers to hedge for this risk. One of the hedging strategies is Delta hedging. Another strategy is Delta-Gamma hedging, or even Delta-Gamma-Vega hedging. However, the last is problematic for the Black-Scholes model as this model assumes a constant volatility. This article focuses on Delta-hedging which is the basis of the other hedging strategies.

The idea of Delta-hedging is that by having a position in an option, you as an investor are exposed to risk. As explained, this risk lies within the unpredictable movement of the underlying of the option, in this case the stock price. The goal of Delta-hedging is to open a hedge position that opposes this risk exposure. See the following formula for clarification:

Option position (with risk exposure) + hedge position = Delta neutrality

An important aspect of Delta-hedging is that it is a **dynamic** hedging strategy. But what does this mean? Unfortunately, the hedging position needs to be adjusted over time. The reason is that locally the value of the hedge portfolio is equal to the call price, but if the underlying of the option, let's say a stock, moves then they are locally not equal anymore meaning that the value of the hedge portfolio needs to be adjusted. This is shown in Figure 3 below. The curved line is the call option price function and the linear line is the hedge portfolio value. The tangent is the point where the curved option price function is equal to the linear hedge portfolio. It shows that when the stock price drops or increases, the value of the call option is not equal to the value of the hedge portfolio anymore. Hence, the hedge portfolio needs to be readjusted.

Figure 3. *Option Price vs. Hedge Portfolio Value*



Source: *Hull J.C. (2022), Options, Futures and other Derivatives 11th edition*

In short, it is not good enough to take a hedging position at initiation of the option position and keep it. This has two reasons:

1. The time to maturity of the option becomes smaller
2. The stock price changes randomly

As the stock price moves constantly, so does the call option price and the value of the hedge portfolio. This means that the readjustment of the hedge portfolio needs to be done many times which leads to two problems:

1. We cannot trade in continuous time as exchanges also close, so we can only trade in discrete time intervals.
2. Besides, trading incurs trading costs. By executing many transactions these costs increase drastically. Hence, investors/asset managers change their hedging positions in discrete time. For example two times a month.

Another interesting hedging strategy is the Delta-Gamma-hedging strategy. For this strategy there is less need for continuous rebalancing. As this is a bit more complicated, we focus in this paper only on Delta hedging. But, if interested we are happy to give insights or explanations if you get into contact with us.

In short, the updating/re-balancing of the hedging portfolio happens in discrete time rises issues. To give better insight into Delta-hedging we provide the following example for you.

Example Delta hedging

An asset manager sells 100 call options that mature in six months and have an exercise price of 100 EUR. The current stock price (the underlying of the option) is 110 EUR, the volatility of the stock is 25%. The risk-free interest rate is 10%. How does the bank achieve a Delta neutral position?

The price of an option can be calculated using the Black Scholes model. This is a continuous time model to calculate the price of an option. We will not get into too much detail how the Black Scholes model works, but focus on the calculation of Delta neutrality. The model calculates the Delta as parameter in the model, hence this can be used to calculate the hedge position for Delta neutrality.

- The Delta of the Black Scholes model, i.e. N(d1) is 0.8186.
- The price of the option using the BS model is: $C_0 = 16.9629$ EUR
- Delta of 100 call options short: -100*0.8186 = -81.86 (-100 as we sell 100 call options)

Step 1. Calculate hedge position for Delta neutrality

Formula for Delta neutrality: Option position + hedge position (π^S) = Delta neutrality

Starting point: position in the call options with strike price of 100 EUR is -100.
Portfolio must be Delta neutral: -100*0.8186 + $\pi^S = 0$ → so $\pi^S = 81.86$.

So, buy 81.86 amount of stocks (the underlying of the option) to be Delta neutral.

Step 2. Calculate financing of the position

By selling 100 calls we earn: 1696.29 EUR in total. However, we need to have 81.86 *110 EUR to be able to buy the hedge position in the stocks. Thus, invest 100*16.9629 – 81.86*110 into the money market account. This is equal to -7308.31. This means that we need to borrow 7308.31 EUR against the risk free rate in order to finance this hedging strategy.